

Revisiting a Tire Wheel Traction Model Using a Modified Width of the Tire-ground Contact Area

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Abstract

Over the years a semi-empiric model describing the interaction between the agricultural tire wheel and terrain was developed and improved. The model was used to predict the traction force and traction efficiency of the driving wheel, based on the formulae given in the ASAE D497.7 standard. The results provided by the model were validated by experimental data based on a goodness-of-fit analysis. For all the previous models the goodness-of-fit analysis has proved that the theoretical results were very well correlated with the experimental data for the traction force (values of the Pearson coefficient r^2 exceeding 0.9), while less reliable results were obtained for the traction efficiency (values of the Pearson coefficient r^2 comprised between 0.20 and 0.65, depending on the geometry taken into account for the shape of the tire cross-section). In order to improve the goodness-of-fit between the model data and the experimental data in the present study three models for the tire-ground interaction were considered: the initial one, developed earlier, which took into account a constant width of the tire, and two modified models, based on an elliptical shape of the tire cross-section, with the width of the tire-ground contact patch smaller than the tire cross-section width (major axis of the ellipse which defines the shape of the cross-section). In the first of these two models the sheared area in the tire-ground interface has varied with the travel reduction of the wheel, while for the second one the shear area was considered constant. Based on the goodness-of-fit analysis it was concluded that the constant shear area model provided the best results, with the Pearson correlation coefficient significantly improved for the traction efficiency ($r^2 = 0.838$), while preserving a high value for the traction force ($r^2 = 0.896$). The model could provide reliable results regarding the traction force and traction efficiency, in certain soil conditions, thus removing the need for experimental tests.

Keywords

shear area, super-ellipse, traction force, traction efficiency

1. Introduction

The use of tractor simulation and prediction models is an essentially low-cost approach for evaluating the significance of different factors affecting the actual tractor operation. Under these circumstances, traction prediction modelling has been driven by the fact that the tire-soil interface is the primary cause of low traction efficiency (estimated to be on the order of 60% on farmland, for transmission efficiency of nearly 90%) [1], without having to build physical prototypes or perform numerous field tests.

The tire-soil interaction has been studied by numerous authors in the attempt to develop traction models for the agricultural tractor driving wheels. The basis for the traction models was established by Bekker in 1956 [2, 3] by developing the basic theory of the wheel-soil interaction. Such models are based on empirical, semi-empirical or analytical methods. Tiwari et al. [4] emphasized some of the difficulties limiting the widespread use of analytical models, including the complex tire-soil interaction,

Proceedings of HAICTA 2022, September 22–25, 2022, Athens, Greece

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CEUR Workshop Proceedings (CEUR-WS.org)

based on a large number of tire and soil parameters. Semi-empirical models are based on the vertical deformation of the soil and on the shear deformation of the soil under a traction device. Empirical models are simpler than analytical and semi-empirical models; however, their applicability is limited to cases in which the service and experimental conditions used to develop the model are similar [5].

Dimensional analysis has also been used in order to develop traction models [6].

Semi-empirical models for wheel-soil interaction, based on Bekker's theory [6], offer a valid framework for modelling the traction performance of the tractor-soil system [7].

The shape of the tire-soil contact area directly affects the traction performance of the driving wheel and numerous algorithms for estimating the contact area of traction tires on agricultural ground were developed over time. The geometry and mechanical properties of the tires should be considered when developing models for the traction of agricultural vehicles [8], but not all the models take into account the tire volume and tire stiffness [9].

In this context, the present paper uses a previously developed traction model [10], further enhancing it by taking into account the geometry and deformation of the tire cross-section in the tire-soil contact area and also a modified width of the contact patch. The aim of this approach was to obtain a better goodness-of-fit between the predicted values of the traction efficiency and the experimental ones.

The paper contains the following sections: "Materials and methods", where the theoretical basis for the models are based are presented, "Assessment of the models", where the comparative results between models data and experimental data are presented, and the "Conclusions" section.

2. Materials and Methods

The model for the tire-soil interaction was based on the schematics presented in Figure 1a [10], assuming that, under the vertical load G , the radius of the tire, in the contact area, increases from r_0 to r_d , while the tire sinks into the soil to the depth z_c .

The shape of the tire-soil contact patch is considered to be a super ellipse (Figure 1b) [11].

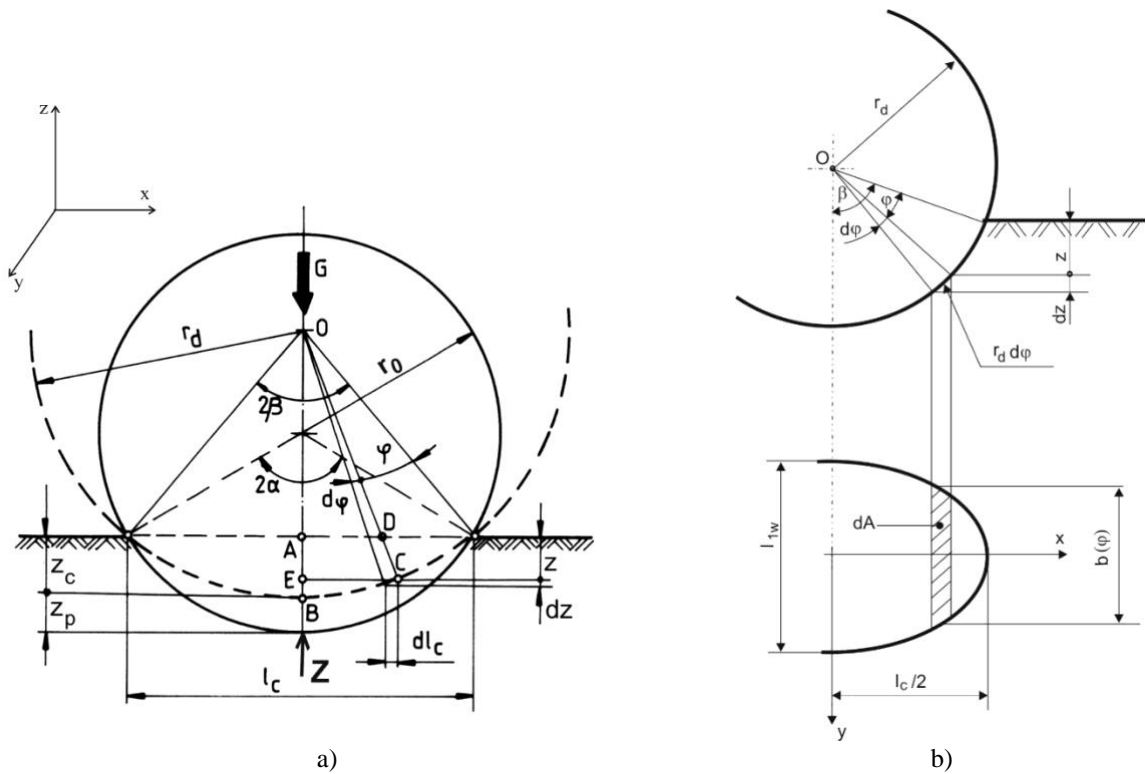


Figure 1: Schematics of the model [10]

a) tire and soil deformation; b) contact patch

z_p -tire deflection under load; z_c -tire sinkage into the soil; l_c -length of the contact patch; l_{1w} -width of the contact patch.

The length of the contact patch (major axis of the contact super ellipse, l_c) is given as:

$$l_c = 2 \cdot r_d \cdot \sin \beta = 2 \cdot r_0 \cdot \sin \alpha. \quad (1)$$

Assuming that the tire is perfectly elastic we get:

$$G = Z = q_p \cdot \Delta V, \quad (2)$$

where q_p is the volume stiffness of the tire and ΔV_p is the variation of the tire volume in the tire-ground contact zone.

In order to evaluate the variation of the tire volume, the initial model [10] assumed that the width of the tire footprint was equal to the width b of the tire (Figure 2). In the upgraded model it was assumed that the transversal cross-section of the tire is an ellipse [12], as shown in Figure 3. With no vertical load, the major axis of the cross-section is b (tire width) and the minor axis is h (section height); under load, the minor axis of the section decreases to $h - z_p$ and the major axis increases to l_w . The value of l_w was calculated assuming that the perimeter of elliptical cross-section remains the same for the initial and final shape:

$$l_w = \sqrt{b^2 + 2 \cdot h \cdot z_p - z_p^2}. \quad (3)$$

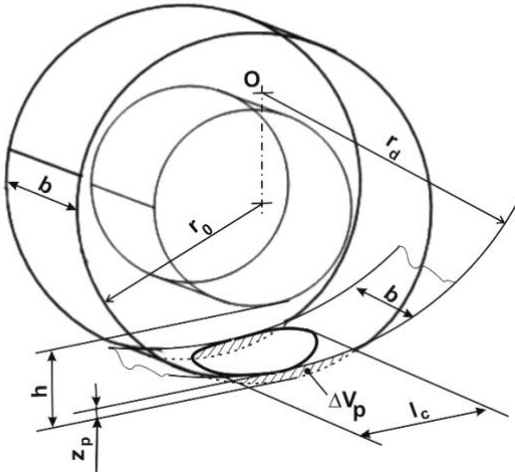


Figure 2: Tire deformation in the contact area (initial model)

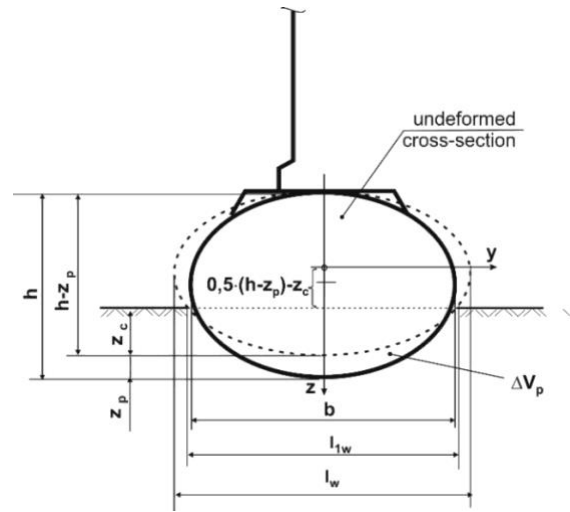


Figure 3: Deformation of the tire cross-section in the contact area (upgraded model)

According to Figures 1b and 3 the width of the tire-soil contact area (minor axis of the super ellipse) is $l_{1w} < l_w$:

$$l_{1w} = l_w \cdot \sqrt{1 - \frac{(h - z_p - 2 \cdot z_c)^2}{(h - z_p)^2}}. \quad (4)$$

The tire volume change ΔV_p was calculated as:

$$\Delta V_p = 0.5 \cdot \pi \cdot [\alpha \cdot r_0 \cdot b \cdot h - \beta \cdot r_d \cdot l_w \cdot (h - z_p)]. \quad (5)$$

The pressure-sinkage relationship [2]:

$$p = k \cdot z^n \quad (6)$$

was applied for the case of the tire under load, finally leading to:

$$G = k \cdot \int_0^{2\beta} r_d^{n+1} \cdot [\cos(\beta - \varphi) - \cos \beta]^n \cdot b(\varphi) \cdot \cos(\beta - \varphi) \cdot d\varphi, \quad (7)$$

The assumption that the tire is perfectly elastic (eq. 2, 5 and 7) led to the following equation:

$$k \cdot \int_0^{2\beta} b(\varphi) \cdot r_d^{n+1} \cdot [\cos(\beta - \varphi) - \cos \beta]^n \cdot \cos(\beta - \varphi) \cdot d\varphi = q_p \cdot 0.5 \cdot \pi \cdot [\alpha \cdot r_0 \cdot b \cdot h - \beta \cdot r_d \cdot l_w \cdot (h - z_p)] \quad (8)$$

where:

$$b(\varphi) = 2 \cdot y = \sqrt[k]{\frac{I_c^2 \cdot I_{1w}^k - [2 \cdot I_{1w} \cdot r_d \cdot \sin(\beta - \varphi)]^k}{I_c^k}} \quad (9)$$

The schematics shown in Figure 1a also led us to the following equations:

$$z_c = r_o - z_p - r_o \cdot \cos \beta \quad (10)$$

$$z_p = r_o \cdot (1 - \cos \alpha) - r_d \cdot (1 - \cos \beta) \quad (11)$$

The system consisting of equations (1), (8), (10) and (11) was solved in an iterative process, using a computer program based on the flowchart shown in Figure 4.

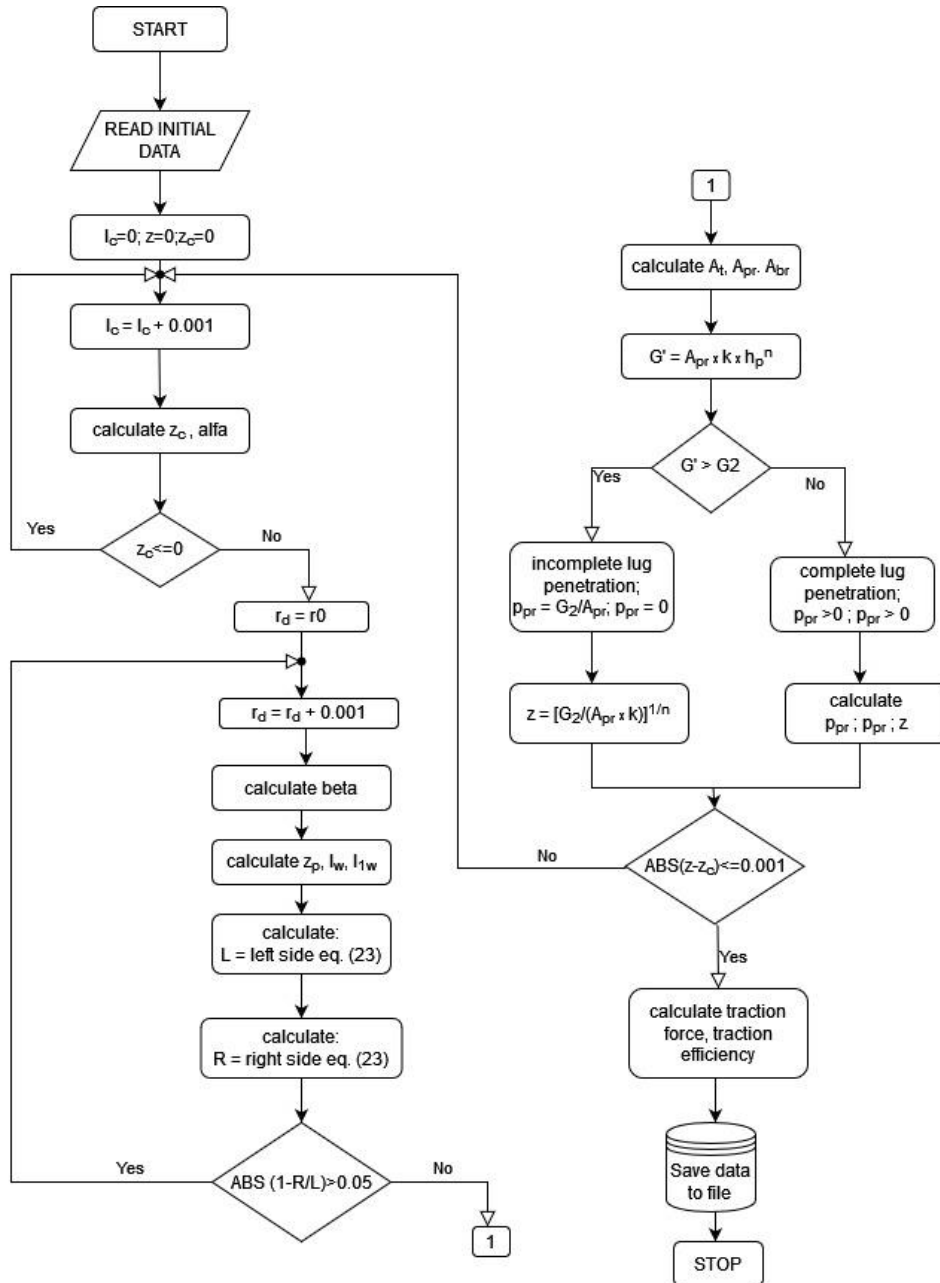


Figure 4: Flowchart of the computer program [10]

The computer program was written in MS QuickBasic and Figure 5 presents a partial view of the program listing and the results window provided by the program.

The area of the tire-ground contact patch was calculated as (Figure 1b):

$$A_t = 2 \cdot \int_0^{l_c/2} b(\varphi) \cdot d\varphi \quad (12)$$

where $b(\varphi)$ is given by equation (9).

The traction force developed by the tire depends on the maximum shear stress which may be achieved in the tire-soil interface, given by the Mohr-Coulomb equation [13]:

$$\tau_{max} = c + p \cdot \tan \gamma \quad (13)$$

where c is soil cohesion [kPa], p is the vertical pressure [kPa] and γ is the internal friction angle.

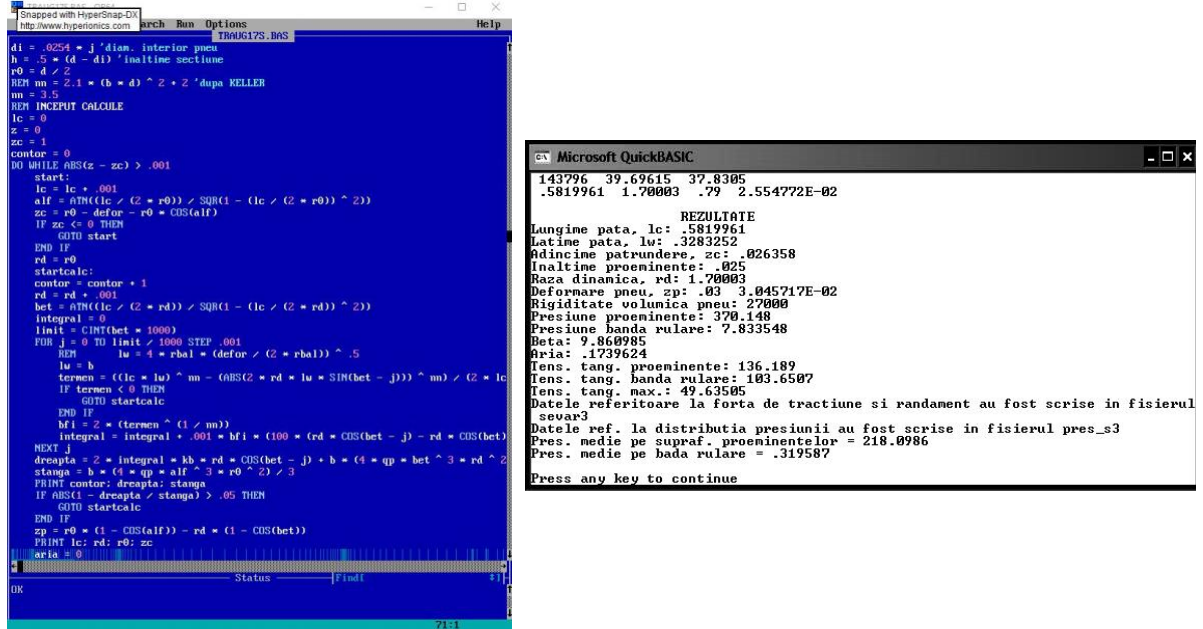


Figure 5: Program listing and results window

The maximum shear stress τ_{max} is not immediately available at the beginning of the contact area, but is reached asymptotically, according to the Janosi and Hanamoto equation [10]:

$$\tau = \tau_{max} \cdot \left(1 - e^{-\frac{J}{K}} \right) \quad (14)$$

where K is the soil shear deformation modulus and J is the shear displacement.

Shear displacement is obtained by integration of slip velocity along the contact patch [14]:

$$J = r_d \cdot [2 \cdot \beta - (1-s) \cdot \sin 2\beta] \quad (15)$$

where s is the wheel slip (wheel travel reduction).

Some authors [15] took into account the fact that the sheared area in the tire-ground interface varies with the travel reduction of the wheel, s :

$$A_{sh} = A_t \cdot [1 - (1-s) \cdot e^{-t}] \quad (16)$$

This assumption was also investigated in our work; as a result, the maximum traction force provided by the wheel, considered as the product of shear stress and shear area, was calculated as:

- $F_t = \tau \cdot A_t$, when the entire contact area was considered;
- $F_t = \tau \cdot A_{sh}$, when the hypothesis of the variable shear area was considered.

The net traction force and traction efficiency were calculated using the formulae given by the ASAE D497.7 standard [16]:

$$F_N = F_t - R_r \quad (17)$$

$$\eta_{tr} = (1-s) \cdot (1 - R_r / F_t) \quad (18)$$

where R_r is the rolling resistance of the wheel, calculated with the help of the wheel numeric, B_n , and cone index, CI [10].

3. Assessment of the Models

Field experiments were performed in order to assess the models. A ploughing equipment, composed of the Romanian U-650 tractor and the variable width P2V plough was used for the tests.

The tractor was equipped with the adequate implements for measuring the traction force and wheel slip [10].

Table 1 presents the main features of the driving wheel and tire; soil characteristics for the test field are shown in Table 2.

The maximum wheel slip during ploughing was 30% and different traction forces were achieved by modifying the working width of the plough.

The experimental data were compared with the data predicted by:

- the initial model [10];
- the upgraded model, based on the variable shear area (given by eq. 16);
- the upgraded model, based on the constant shear area (given by eq. 12).

Table 1.

Characteristics of the U-650 tractor and drive wheels

Item	Value
Load on the driving wheel [kN]	11.75
Type of tire	14.00 – 38
Overall diameter of tire [m]	1.58
Tire width [m]	0.367
Lug width [m]	0.04
Lug length [m]	0.24
Lug height [m]	0.025
Distance between lugs [m]	0.195
Transversal radius of the under tread [m]	0.300

Table 2

Characteristics of the test soil

Item	Value
Soil deformation modulus, K [m]	0.05
Coefficients for the sinkage equation	k
	n
Soil cohesion, c [kPa]	25
Angle of internal friction, ϕ [°]	32
Cone penetrometer index, CI [kPa]	970

A goodness-of-fit analysis was performed in order to compare the predicted data with the experimental ones; the analysis took into account the following criteria [17]:

- percentage of points within the 95% confidence interval of data (Pw95CI);
- mean absolute deviation (MAD);
- root mean squared deviation (RMSD);
- mean scaled absolute deviation (MSAD);
- Pearson correlation coefficient r^2 .

The goodness-of-fit analysis was performed for both the traction force and the traction efficiency.

As mentioned before, three models were evaluated: the initial one, the one assuming that the tire cross-section is an ellipse and that the tire-soil shear area is affected by wheel slip and the one also assuming an elliptical shape of the tire cross-section but which took into account a constant shear area.

The results characterizing the tire-ground contact are summarized in Table 3. These data show that, due to the fact that the width of the contact patch was considered to be equal with the tire width in the initial model, that model has predicted a larger contact area compared with the other two models; as a

result, the maximum shear stress at the tire-ground interface was lower for the initial model. The dynamic radius obtained for this model was greater and therefore led to a greater length of the contact patch.

Table 3

Parameters for the tire-ground contact

Item	Tire-ground model		
	constant width of tire (Rosca, 2014)	deformable cross-section, variable shear area	deformable cross-section, constant shear area
l_c [m]	0.530	0.534	0.534
l_w [m]	0.367	0.384	0.384
l_{1w} [m]	-	0.222	0.222
z_c [m]	0.0271	0.0275	0.0275
r_d [m]	1.400	1.257	1.257
A_t [m ²]	0.145	0.1086	0.1086
τ_{max} [kPa]	53.3	61.2	61.2

There were no differences between the upgraded models as far as the tire-ground contact geometry was concerned because the effect of wheel slip on the shear area was taken into account at a later stage (see Figure 4), for the prediction of traction force and traction efficiency.

The results referring to the traction force are presented in Figure 6 and Table 4.

The results depicted in Figure 6 prove that the initial model and the one based on the elliptic shape of the tire cross-section and a constant shear area have provided quite similar predictions as long as wheel slip did not exceed 25%, while the model based on the on the elliptic shape of the tire cross-section and a variable shear area has provided significantly lower values for the traction force.

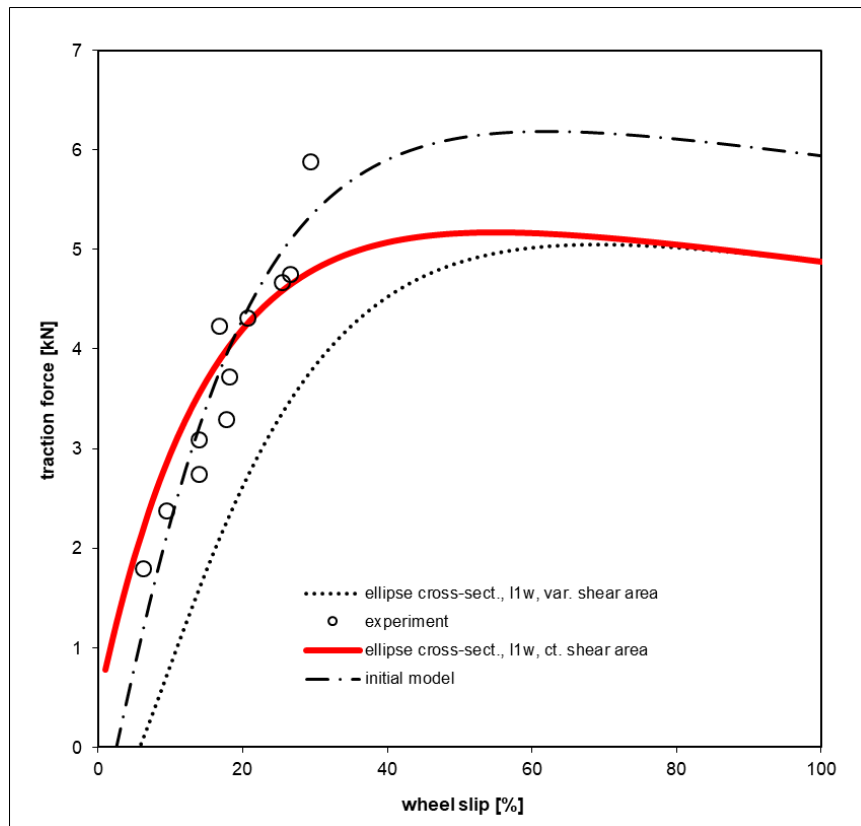


Figure 6: Predicted and experimental data for traction force

The results of the goodness-of-fit analysis, shown in Table 4, have confirmed that the model based on the elliptical shape of the tire cross-section and variable shear area has provided the most inaccurate results, as it predicted much lower values of the traction force than the ones obtained from the experimental tests. Despite the achievement the highest value of the Pearson correlation coefficient, all the other items were unfavorable: thus, the value of MSAD criterion means that, on average, the model was 8.817 standard errors off from the experimental data; the percentage of model predictions that lie within the 95% confidence interval of each corresponding experimental data point was only 11.1%; the mean absolute deviation between each model point and the corresponding experimental point was 1.6408 (the highest value of all the models taken into account).

On the other hand, for the model based on the constant shear area, although the value of r^2 was lower (but only by 2.7% in comparison with the initial model), the values of the all the other items taken into account within the goodness-of-fit analysis (Pw96CI = 88.9%; MSAD = 1.184 etc.) led us to the conclusion that this model was the most accurate for describing the tire-soil interaction.

Table 4
Results of the goodness-of-fit analysis for traction force

Item	Tire-ground model		
	constant width of tire (Rosca, 2014)	deformable cross-section, variable shear area	deformable cross-section, constant shear area
r^2	0.921	0.981	0.896
MAD	0.3353	1.6408	0.3981
MSAD	3.116	8.817	1.184
Pw95CI	66.7	11.1	88.9
RMSD	0.4379	1.685	0.4952

Figure 7 and Table 5 present the results referring to the traction efficiency. As it can be concluded from Table 5, the main problem of the initial model was that it provided relatively poor results regarding the traction efficiency ($r^2 = 0.203$) and this was the driving force that steered us towards the improvement of that specific model.

The data presented in Figure 7 clearly show that the model based on the elliptical shape of the tire cross-section and constant shear area was the most adequate for describing the tire-soil interaction process in terms of traction efficiency, while the values predicted by the model based on variable shear area were significantly lower than the corresponding experimental data.

The results shown in Table 5 prove that the model based on a constant shear area has predicted more reliable results than the initial one due to the following:

- a significantly higher value of the Pearson correlation coefficient in comparison with the initial one (0.838 vs. 0.203)
- all the predicted data points were within the 95% confidence interval of the corresponding experimental data (Pw95CI = 100);
- all the other items taken into account for the goodness-of-fit analysis had lower values than the ones recorded for both the initial model and the one using a variable shear area.

4. Conclusions

Over the years a semi-empiric model describing the interaction between the agricultural tire wheel and terrain was developed and improved. The model was used to predict the traction force and traction efficiency of the driving wheel.

In order to improve the goodness-of-fit between model and experimental a new geometry of the tire cross-section was taken into account, assuming that the tire has an elliptical shape, which deforms under vertical load while still preserving the elliptic shape.

The data predicted by the original model were compared with the ones predicted by the modified model and with experimental data, collected during ploughing.

The best results were obtained when the constant tire-soil shear area was considered; the Pearson correlation coefficient has significantly improved for the traction efficiency ($r^2 = 0.838$), while preserving a high value for the traction force ($r^2 = 0.896$).

The percentage of model-predicted data points which fall within the 95% confidence interval of experimental data has increased to 88.9% for the traction force and to 100% for the traction efficiency.

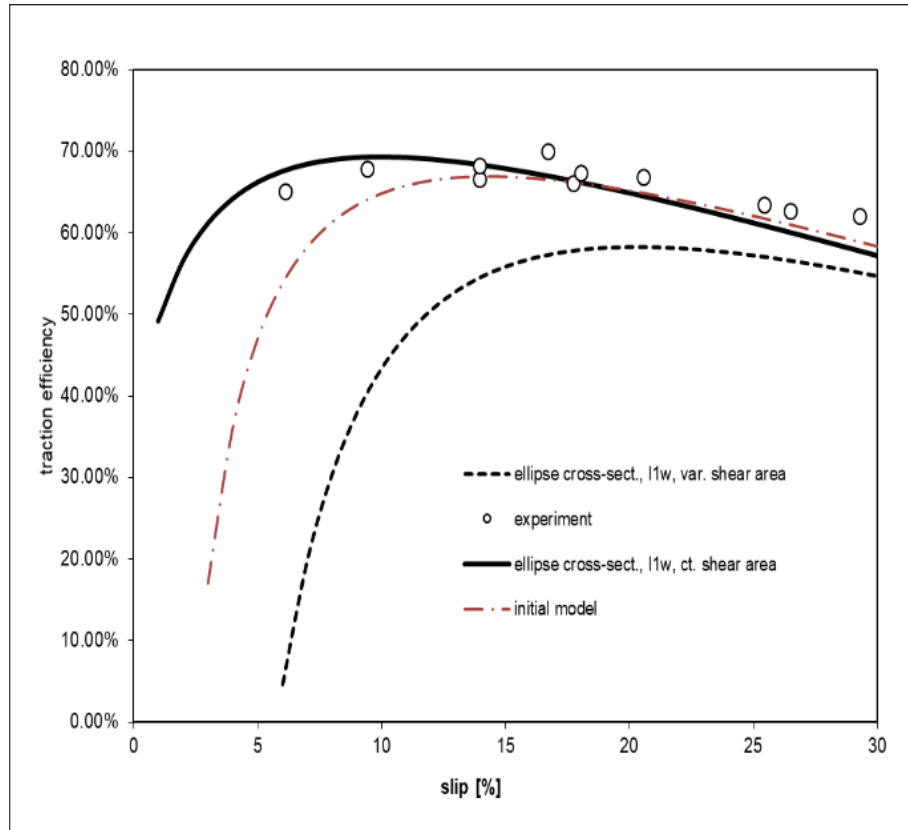


Figure 7: Predicted and experimental data for traction efficiency

Table 5

Results of the goodness-of-fit analysis for traction efficiency

Item	Tire-ground model type		
	constant width of tire (Rosca, 2014)	deformable cross-section, variable shear area	deformable cross-section, constant shear area
r^2	0.203	0.726	0.838
MAD	0.0569	0.0953	0.0198
MSAD	5.147	1.966	0.309
Pw95CI	55.6	88.9	100
RMSD	0.075	0.1279	0.0231

It should be emphasized that, in this stage, the presented results are valid only for the specific soil conditions from the test field. More experiments on terrains with different soil characteristics are needed in order to extend the validity of the upgraded model, based on the constant shear area.

One promising direction seems to be the use of a rheological model for the shear deformation of soil, but this is still a difficult task as no reliable data regarding the rheological properties of different soils are available [18].

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