Rational Wavelet Transform with Reducible Rational Dilation Factor

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Abstract. Wavelet analysis is very effectively used in analysis of different types of data. Mostly often dyadic wavelet transforms are used. But non-dyadic wavelet transforms allow more accurate determination of data features. Commonly used value of dilation factor for rational wavelet transforms is an irreducible fraction. In this paper we will show on an example that for reducible fraction as a dilation factor perfect reconstruction condition is satisfied.

Keywords: Wavelet Transform, Non-Dyadic Wavelet, Dilation Factor.

1 Introduction

1.1 Wavelet Transform

Wavelet transform (WT) is a very powerful tool for analyzing the data of different nature. Unlike Fourier analysis WT gives as a result time-frequency representation of source signal.

Wavelet analysis has shown its efficiency in various areas, such as image and video processing, data compression and noise reduction, solving of partial differential equations, speech recognition, processing of Electroencephalography (EEG), Electromyography (EMG), Electrocardiography (ECG) signals, etc. [1-4]

1.2 Dyadic and Non-dyadic Wavelet Transform

Discrete wavelet transform is characterized by its dilation factor. It was shown [5] that as a value of dilation factor any real number greater than one can be taken.

If dilation factor equals 2 the discrete WT is called dyadic, otherwise non-dyadic. Usually dyadic WT is used due to its simple and effective implementation. But in the case when signal singularities will be located in adjoined frequency intervals the results of dyadic WT will not be eligible. Thus, it will be better to select frequency intervals in a way that they would cover such singularities. For example, non-dyadic WT are more suitable for flexible decompositions of the data [6], non-dyadic scale ratios [7], non-dyadic frequency divisions [8], or constructing non-tensor-based multi-D wavelets with coset sum method [9].

Various approaches to non-dyadic WT were proposed by different authors.

Bratteli and Jorgensen [10] proposed a method based on the construction of the set mapping to Kunzh's algebra representation. In their approach task of non-dyadic WT filters choice reduces to unitary matrix construction. As a dilation factor only a natural number greater than one can be selected.

Pollock and Cascio [8] proposed a method for construction of packet WT with a possibility of a dilation factor choice through each level of decomposition. They generalized dyadic WT packet technique, where each frequency range at each step of wavelet decomposition is divided into two intervals. Main idea was to divide range into p intervals, where p is an arbitrary prime number.

There can be cases where integer dilation factor is not enough. Auscher [11] gave the formal definition of a rational multiresolution analysis and specified the method of corresponding orthogonal wavelet bases construction. Using his ideas, Baussard, Nicolier and Truchetet [12] developed a fast pyramidal algorithm for WT coefficients construction in a case of a rational dilation factor. Thus, they gave a generalization of the Malla algorithm for dyadic case.

Also, there are some works that deal with using of irrational dilation factors. One of the first was Feauveau [13], where the value of dilation factor was $\sqrt{2}$.

1.3 Rational Wavelet Transform

The most general and effective approach for non-dyadic WT is rational multiresolution analysis proposed in [11] and [12].

In this approach dilation factor N is equal to a rational number p/q. At each level of wavelet decomposition, we get one approximation component and (p – q) detail components. Rational multiresolution analysis filters construction is carried out in the frequency domain. Wavelet decomposition pyramidal algorithm with a rational dilation factor is also transferred to the frequency domain.

2 Problem Formulation

The main advantage of non-dyadic wavelet transform is that it can provide more precise separation of signal components. Very often the irreducible dilation factor is used for such decomposition. Commonly its value is taken 3/2.

The purpose of this work is to show that as value of the dilation factor for rational wavelet transform a reducible fraction 6/4 can be taken and this will allow getting the more precise detection of signal singularities. Also, authors will show that perfect reconstruction condition is satisfied for this case.

3 Problem Solution

3.1 Conditions for Building Filters

In order to build filters for rational wavelet transform with dilation factor 6/4 we will use the approach described in [5] for case 3/2.

We have function

$$\varphi \in V_0 \subset V_{-1} = \overline{Span\left\{\varphi\left(\frac{6}{4}\cdot -n\right)\right\}}$$

so, that

$$\varphi(x) = \sqrt{\tfrac{6}{4}}\textstyle\sum_n h_n^0 \varphi\left(\tfrac{6}{4}x - n\right) \quad (1)$$

$$\varphi(x-4l) = \sqrt{\frac{6}{4}}\sum_n h_n^0 \varphi\left(\frac{6}{4}(x-4l) - n\right) = \sqrt{\frac{6}{4}}\sum_n h_n^0 \varphi\left(\frac{6}{4}x - 6l - n\right) =$$

$$= \sqrt{\frac{6}{4}}\sum_k h_{k-6l}^0 \varphi\left(\frac{6}{4}x - k\right)$$

This set of functions has to be orthonormal

$$\delta_{0l} = \langle \varphi(x), \varphi(x-4l)\rangle = \int_{\mathbb{R}} \varphi(x)\cdot\overline{\varphi(x-4l)}\,dx =$$

$$= \int_{\mathbb{R}} \left(\sqrt{\frac{6}{4}}\sum_n h_n^0 \varphi\left(\frac{6}{4}x - n\right)\right)\left(\sqrt{\frac{6}{4}}\sum_k \overline{h_{k-6l}^0 \varphi\left(\frac{6}{4}x - k\right)}\right)dx$$

$$= \frac{6}{4}\sum_n\sum_k h_n^0 \overline{h_{k-6l}^0}\int_{\mathbb{R}} \varphi\left(\frac{6}{4}x - n\right)\cdot\overline{\varphi\left(\frac{6}{4}x - k\right)}\,dx = \sum_n h_n^0 \overline{h_{n-6l}^0}$$

So, orthonormality of the $\varphi(\cdot - 4l)$ implies the condition

$$\sum_n h_n^0 \overline{h_{n-6l}^0} = \delta_{0l}$$

On the other hand, $\varphi(x-1)$ is also in V_0, and therefore can also be written as

$$\varphi(x-1) = \sqrt{\tfrac{6}{4}}\textstyle\sum_n h_n^1 \varphi\left(\tfrac{6}{4}x - n\right) \quad (2)$$

$$\varphi(x-4l-1)=\sqrt{\frac{6}{4}}\sum_n h_n^1\varphi\left(\frac{6}{4}(x-4l)-n\right)=\sqrt{\frac{6}{4}}\sum_n h_n^1\varphi\left(\frac{6}{4}x-6l-n\right)=$$

$$=\sqrt{\frac{6}{4}}\sum_k h_{k-6l}^1\varphi\left(\frac{6}{4}x-k\right)$$

Set of functions $\varphi(\cdot -1)$ also has to be orthonormal.

$$\delta_{0l}=\langle\varphi(x-1),\varphi(x-4l-1)\rangle=\int_{\mathbb{R}}\varphi(x-1)\cdot\overline{\varphi(x-4l-1)}\,dx=$$

$$=\int_{\mathbb{R}}\left(\sqrt{\frac{6}{4}}\sum_n h_n^1\varphi\left(\frac{6}{4}x-n\right)\right)\left(\sqrt{\frac{6}{4}}\sum_k \overline{h_{k-6l}^1\varphi\left(\frac{6}{4}x-k\right)}\right)dx$$

$$=\frac{6}{4}\sum_n\sum_k h_n^1\overline{h_{k-6l}^1}\int_{\mathbb{R}}\varphi\left(\frac{6}{4}x-n\right)\cdot\overline{\varphi\left(\frac{6}{4}x-k\right)}\,dx=\sum_n h_n^1\overline{h_{n-6l}^1}$$

Also, it has to be orthogonal to the previous set of functions.

$$0=\langle\varphi(x-1),\varphi(x-4l)\rangle=\int_{\mathbb{R}}\varphi(x-1)\cdot\overline{\varphi(x-4l)}\,dx=$$

$$=\int_{\mathbb{R}}\left(\sqrt{\frac{6}{4}}\sum_n h_n^1\varphi\left(\frac{6}{4}x-n\right)\right)\left(\sqrt{\frac{6}{4}}\sum_k \overline{h_{k-6l}^0\varphi\left(\frac{6}{4}x-k\right)}\right)dx$$

$$=\frac{6}{4}\sum_n\sum_k h_n^1\overline{h_{k-6l}^0}\int_{\mathbb{R}}\varphi\left(\frac{6}{4}x-n\right)\cdot\overline{\varphi\left(\frac{6}{4}x-k\right)}\,dx=\sum_n h_n^1\overline{h_{n-6l}^0}$$

So, orthonormality of the $\varphi(.-4l-1)$ and its orthogonality of the $\varphi(.-4l-1)$ with the respect to $\varphi(.-4l)$ give us such two conditions

$$\sum_n h_n^1\overline{h_{n-6l}^1}=\delta_{0l}$$

$$\sum_n h_n^1\overline{h_{n-6l}^0}=0$$

Repeating the similar operations for functions $\varphi(x-2)$ and $\varphi(x-3)$

$$\varphi(x-2) = \sqrt{\tfrac{6}{4}}\sum_n h_n^2 \varphi\left(\tfrac{6}{4}x - n\right)$$
$$\varphi(x-3) = \sqrt{\tfrac{6}{4}}\sum_n h_n^3 \varphi\left(\tfrac{6}{4}x - n\right) \quad (3)$$

and taking into account orthogonality and orthonormality properties will bring us to the conditions

$$\sum_n h_n^2 \overline{h_{n-6l}^2} = \delta_{0l}$$
$$\sum_n h_n^2 \overline{h_{n-6l}^0} = 0$$
$$\sum_n h_n^2 \overline{h_{n-6l}^1} = 0$$
$$\sum_n h_n^3 \overline{h_{n-6l}^3} = \delta_{0l}$$
$$\sum_n h_n^3 \overline{h_{n-6l}^0} =$$
$$\sum_n h_n^3 \overline{h_{n-6l}^1} =$$
$$\sum_n h_n^3 \overline{h_{n-6l}^2} =$$

Now, we will introduce auxiliary functions. Let's denote by $\hat{\varphi}(\omega)$ the Fourier transform of function $\varphi(x)$. Then, applying the Fourier transform to (1), we will get

$$\hat{\varphi}(\omega) = \sqrt{\frac{6}{4}}\sum_n \left(h_n^0 \frac{4}{6}\hat{\varphi}\left(\frac{4}{6}\omega\right) e^{-i\frac{4}{6}n\omega}\right) = \left(\sqrt{\frac{4}{6}}\sum_n h_n^0 e^{-i\frac{4}{6}n\omega}\right) \cdot \hat{\varphi}\left(\frac{4}{6}\omega\right)$$

We can rewrite last expression as

$$\hat{\varphi}(\omega) = m_0^0\left(\tfrac{4}{6}\omega\right) \cdot \hat{\varphi}\left(\tfrac{4}{6}\omega\right) \quad (4)$$

where $m_0^0(\omega)$ is defined as

$$m_0^0(\omega) \equiv \sqrt{\frac{4}{6}}\sum_n h_n^0 e^{-in\omega}$$

In the same way we can define the function $m_0^1(\omega)$. Applying the Fourier transform to (2) will give us

$$\hat{\varphi}(\omega)e^{-i\omega} = \sqrt{\frac{6}{4}}\sum_n \left(h_n^1 \frac{4}{6}\hat{\varphi}\left(\frac{4}{6}\omega\right)e^{-i\frac{4}{6}n\omega}\right) = \left(\sqrt{\frac{4}{6}}\sum_n h_n^1 e^{-i\frac{4}{6}n\omega}\right)\cdot\hat{\varphi}\left(\frac{4}{6}\omega\right)$$

$$\hat{\varphi}(\omega)e^{-i\omega} = m_0^1\left(\tfrac{4}{6}\omega\right)\cdot\hat{\varphi}\left(\tfrac{4}{6}\omega\right) \tag{5}$$

where

$$m_0^1(\omega) \equiv \sqrt{\frac{4}{6}}\sum_n h_n^1 e^{-in\omega}$$

After applying the same operations to functions (3) we will get

$$\hat{\varphi}(\omega)e^{-i2\omega} = m_0^2\left(\tfrac{4}{6}\omega\right)\cdot\hat{\varphi}\left(\tfrac{4}{6}\omega\right) \tag{6}$$

$$\hat{\varphi}(\omega)e^{-i3\omega} = m_0^3\left(\tfrac{4}{6}\omega\right)\cdot\hat{\varphi}\left(\tfrac{4}{6}\omega\right) \tag{7}$$

where

$$m_0^2(\omega) \equiv \sqrt{\frac{4}{6}}\sum_n h_n^2 e^{-in\omega}$$

$$m_0^3(\omega) \equiv \sqrt{\frac{4}{6}}\sum_n h_n^3 e^{-in\omega}$$

Similar to [5] we can also define two wavelet functions

$$\psi_1(x) = \sqrt{\frac{6}{4}}\sum_n g_n^1\varphi\left(\frac{6}{4}x - n\right)$$

$$\psi_2(x) = \sqrt{\frac{6}{4}}\sum_n g_n^2\varphi\left(\frac{6}{4}x - n\right)$$

Properties of their orthonormality, mutual orthogonality and orthogonality with previously defined functions φ give us the next conditions

$$\sum_n g_n^1\overline{g_{n-6l}^1} = \delta_{0l} \quad \sum_n g_n^2\overline{g_{n-6l}^2} = \delta_{0l}$$

$$\sum_n g_n^1\overline{h_{n-6l}^0} = 0 \quad \sum_n g_n^2\overline{h_{n-6l}^0} = 0$$

$$\sum_n g_n^1 \overline{h_{n-6l}^1} = 0 \quad \sum_n g_n^2 \overline{h_{n-6l}^1} = 0$$

$$\sum_n g_n^1 \overline{h_{n-6l}^2} = 0 \quad \sum_n g_n^2 \overline{h_{n-6l}^2} = 0$$

$$\sum_n g_n^1 \overline{h_{n-6l}^3} = 0 \quad \sum_n g_n^2 \overline{h_{n-6l}^3} = 0$$

$$\sum_n g_n^2 \overline{g_{n-6l}^1} = 0$$

Also, we get another two necessary functions $m_1(\xi)$ and $m_2(\xi)$

$$m_1(\omega) = \sqrt{\frac{4}{6}} \sum_n g_n^1 e^{-in\omega}$$

$$m_2(\omega) = \sqrt{\frac{4}{6}} \sum_n g_n^2 e^{-in\omega}$$

3.2 Perfect Reconstruction Conditions

According to [5], received conditions for filters coefficients are equivalent to the unitarity of matrix

$$\begin{pmatrix} m_0^0(\omega) & m_0^1(\omega) & m_0^2(\omega) & m_0^3(\omega) & m_1(\omega) & m_2(\omega) \\ m_0^0\left(\omega+\frac{\pi}{3}\right) & m_0^1\left(\omega+\frac{\pi}{3}\right) & m_0^2\left(\omega+\frac{\pi}{3}\right) & m_0^3\left(\omega+\frac{\pi}{3}\right) & m_1\left(\omega+\frac{\pi}{3}\right) & m_2\left(\omega+\frac{\pi}{3}\right) \\ m_0^0\left(\omega+\frac{2\pi}{3}\right) & m_0^1\left(\omega+\frac{2\pi}{3}\right) & m_0^2\left(\omega+\frac{2\pi}{3}\right) & m_0^3\left(\omega+\frac{2\pi}{3}\right) & m_1\left(\omega+\frac{2\pi}{3}\right) & m_2\left(\omega+\frac{2\pi}{3}\right) \\ m_0^0\left(\omega+\frac{3\pi}{3}\right) & m_0^1\left(\omega+\frac{3\pi}{3}\right) & m_0^2\left(\omega+\frac{3\pi}{3}\right) & m_0^3\left(\omega+\frac{3\pi}{3}\right) & m_1\left(\omega+\frac{3\pi}{3}\right) & m_2\left(\omega+\frac{3\pi}{3}\right) \\ m_0^0\left(\omega+\frac{4\pi}{3}\right) & m_0^1\left(\omega+\frac{4\pi}{3}\right) & m_0^2\left(\omega+\frac{4\pi}{3}\right) & m_0^3\left(\omega+\frac{4\pi}{3}\right) & m_1\left(\omega+\frac{4\pi}{3}\right) & m_2\left(\omega+\frac{4\pi}{3}\right) \\ m_0^0\left(\omega+\frac{5\pi}{3}\right) & m_0^1\left(\omega+\frac{5\pi}{3}\right) & m_0^2\left(\omega+\frac{5\pi}{3}\right) & m_0^3\left(\omega+\frac{5\pi}{3}\right) & m_1\left(\omega+\frac{5\pi}{3}\right) & m_2\left(\omega+\frac{5\pi}{3}\right) \end{pmatrix} \tag{8}$$

Li [14] proved that perfect reconstruction condition is satisfied for the case of the irreducible dilation factor and showed two examples of wavelet basis.

In [10] the extension of Littlewood-Paley wavelet basis to the rational case was introduced.

Both the scale function and wavelet functions in this case are defined in Fourier domain by next formulas:

$$|\hat{\varphi}(N\omega)| = \begin{cases} 1 & for \quad -\frac{q\cdot\pi}{p} \le \omega < \frac{q\cdot\pi}{p} \\ 0 & elsewhere \end{cases}$$

$$\left|\hat{\psi}^m(N\omega)\right| = \begin{cases} 1 & if \quad \frac{(q+m-1)\cdot\pi}{p} \le |\omega| < \frac{(q+m)\cdot\pi}{p} \\ 0 & elsewhere \end{cases}$$

where $N = \frac{p}{q}$ and $m = 1, \dots, p-q$.

We will consider this wavelet basis for the case of reducible fraction 6/4 and check that perfect reconstruction condition will be satisfied.

If $N = \frac{6}{4}$ then scale function can be taken as

$$\hat{\varphi}\left(\frac{6}{4}\omega\right) = \begin{cases} 1 & if \quad |\omega| \le \frac{4\pi}{6} \\ 0 & elsewhere \end{cases}$$

or

$$\hat{\varphi}(\omega) = \begin{cases} 1 & if \quad |\omega| \le \pi \\ 0 & elsewhere \end{cases}$$

Wavelet functions can be defined as

$$\hat{\psi}^1\left(\frac{6}{4}\omega\right) = \begin{cases} 1 & if \quad \frac{4\pi}{6} \le |\omega| < \frac{5\pi}{6} \\ 0 & elsewhere \end{cases}$$

$$\hat{\psi}^2\left(\frac{6}{4}\omega\right) = \begin{cases} 1 & if \quad \frac{5\pi}{6} \le |\omega| < \pi \\ 0 & elsewhere \end{cases}$$

According to the (4) – (7) we can write

$$m_0^0(\omega) = \frac{\hat{\varphi}\left(\frac{6}{4}\omega\right)}{\hat{\varphi}(\omega)}$$

$$m_0^1(\omega) = \frac{\hat{\varphi}\left(\frac{6}{4}\omega\right)}{\hat{\varphi}(\omega)} \cdot e^{-i\frac{6}{4}\cdot\omega}$$

$$m_0^2(\omega) = \frac{\hat{\varphi}\left(\frac{6}{4}\omega\right)}{\hat{\varphi}(\omega)} \cdot e^{-i\frac{6}{4}\cdot 2\omega}$$

$$m_0^3(\omega) = \frac{\hat{\varphi}\left(\frac{6}{4}\omega\right)}{\hat{\varphi}(\omega)} \cdot e^{-i\frac{6}{4}\cdot 3\omega}$$

Similarly, we get

$$\mathrm{m}_1(\omega) = \frac{\hat{\psi}^1\left(\frac{6}{4}\omega\right)}{\hat{\varphi}(\omega)}$$

$$\mathrm{m}_2(\omega) = \frac{\hat{\psi}^2\left(\frac{6}{4}\omega\right)}{\hat{\varphi}(\omega)}$$

After substituting the above expressions into the matrix (8) direct check allows us to verify that the condition of perfect reconstruction, given in [14], will be also satisfied for the case of reducible fraction 6/4.

3.3 Experimental Results

In order to illustrate that using of the reducible dilation factor can lead to more precise identifying of signal singularities we will take a look at the example which is based on model data.

This mode data is the sum of three sinusoidal signals (see Fig. 1).

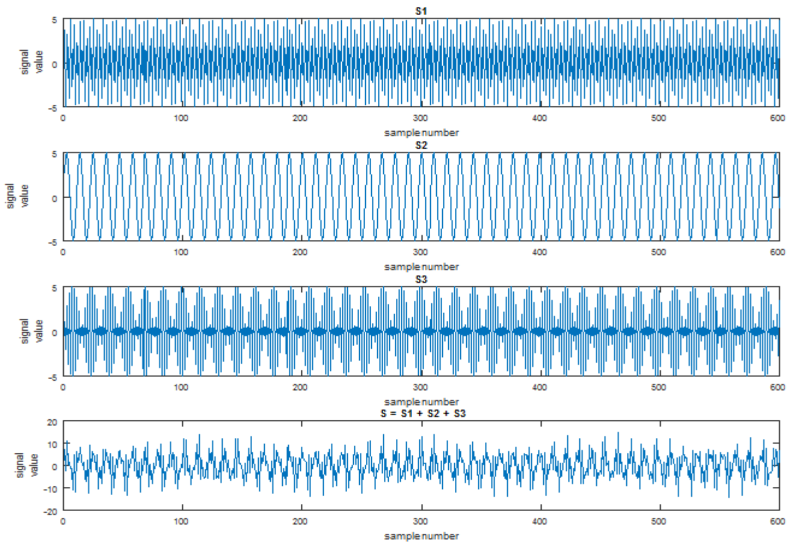

Fig. 1. Model signal

Fourier spectrum of this model signal contains three peaks in frequency domain, that fall into the following frequency ranges (see Fig.2):

- first signal (S1) – from 2π/3 to 5π/6

- second signal (S2) – from 0 to 2π/3
- third signal (S3) – from 5π/6 to π

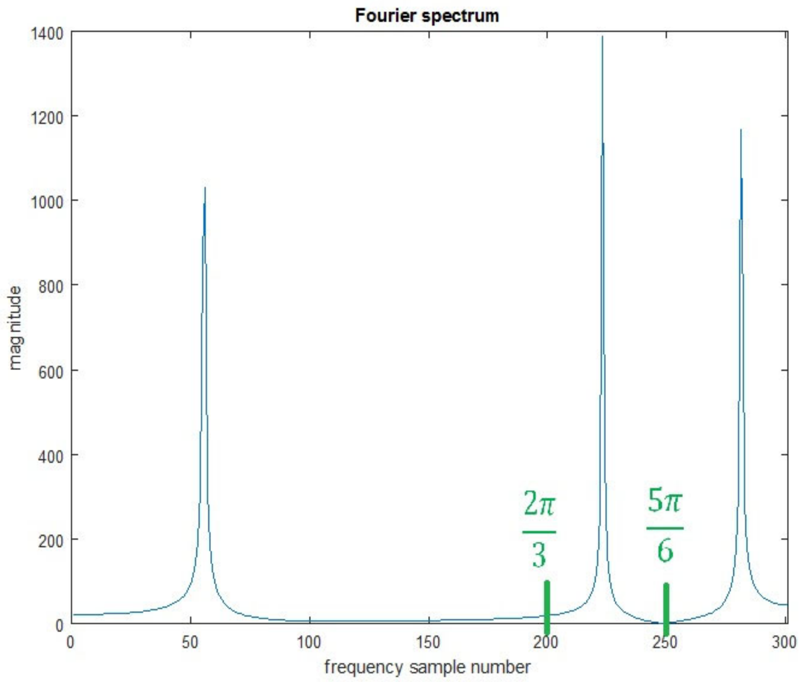

Fig. 2. Fourier spectrum of signal

When using the dilation factor 3/2 frequency range is divided into two parts – from 0 to 2π/3 for approximation component and from 2π/3 to π for detail component. Due to this rational wavelet transform will separate signal S2 (see Fig.3. Approximation component), but signal S1 and S3 will be mixed (see Fig.3. Detail component).

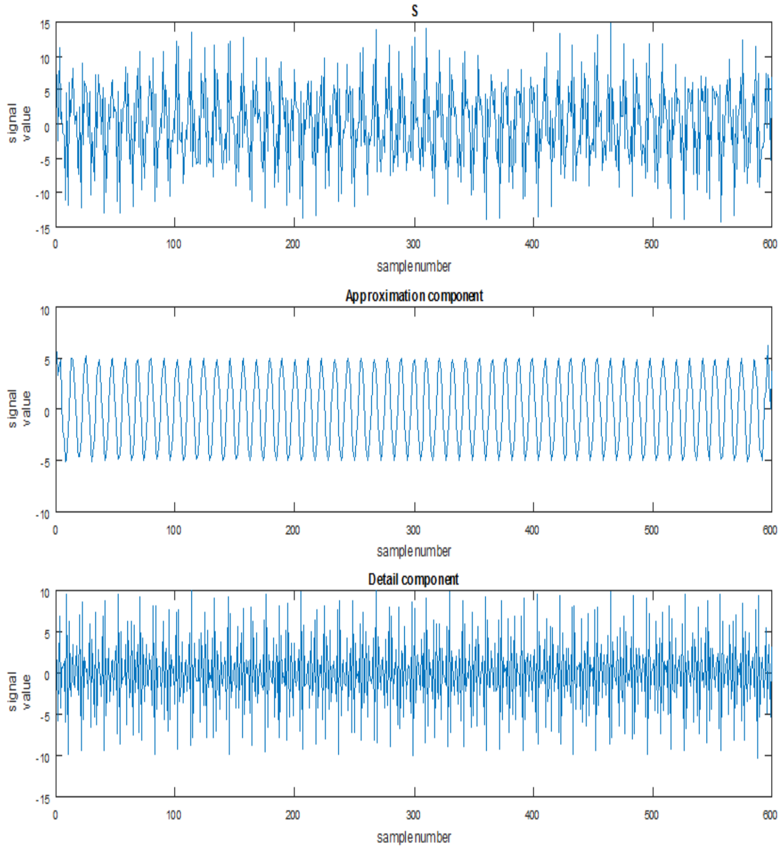

Fig. 3. Decomposition of model signal with dilation factor 3/2

But, if we take dilation factor 6/4 then frequency range is divided into three parts – again from 0 to $2\pi/3$ for approximation component, but now from $2\pi/3$ to $5\pi/6$ for the first detail component and from $5\pi/6$ to π for the second detail component.

Thus, in this case all three source signals will be separated (see Fig. 4).

So, for this model signal the use of dilation factor 6/4 is preferable, since it allows separation of all three components from the original signal.

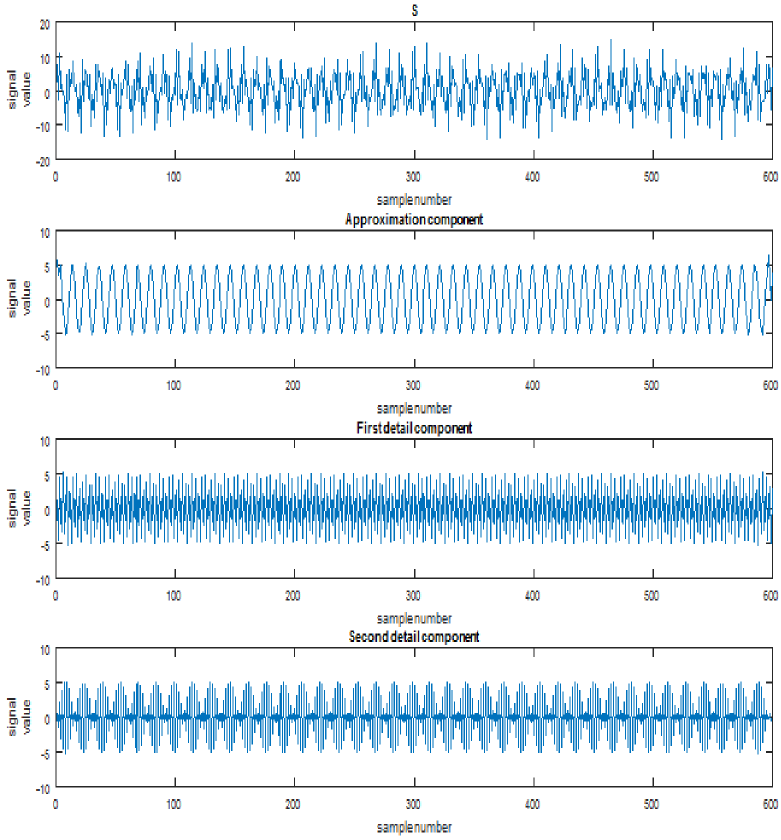

Fig. 4. Decomposition of model signal with dilation factor 6/4

4 Conclusions

Authors proposed to use reducible rational dilation factor 6/4 except most often used 3/2. It was shown that for this value of dilation factor perfect reconstruction conditions are satisfied. An example shows that using dilation factor 6/4 allows more precise separation of signal singularities.

Further researches have to be done in order to generalize the results to the case of arbitrary reducible dilation factor.

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